**7. FOURIER SERIES**

**PERIODIC FUNCTION:** A function is called a periodic function, if is defined for all real except possibly at some points, and if there is some positive number , called a period of , such that,

The smallest positive period is often called the fundamental period.

**FOURIER SERIES:**

Fourier series is representation of a non-sinusoidal periodic function sum of sinusoids.

Consider a periodic function with periodicity , the trigonometric Fourier series (TFS) of is given by,

|  |  |
| --- | --- |
|  | Where, = Trigonometric Fourier series coefficient  = Average value of  = Term independent of “x”  = Constant term |
|  |
|  |  |

|  |  |
| --- | --- |
| **EVEN FUNCTION:**  A function is said to be an even function if | **ODD FUNCTION:**  A function is said to be an even function if |
| Rotation about Y-axis | Rotation about X-axis |

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| **SYMMETRY** | **CONDITION** |  |  |  | **PROPERTY** |
| **EVEN** |  | YES | YES | 0 | Cosine Term only |
| **ODD** |  | 0 | 0 | YES | Sine Term only |

**HALF RANGE SERIES:**

* Half Range Cosine series of in range *(0, l)* is same as Even Fourier series of .
* Half Range Sine series of in range *(0, l)* is same as Odd Fourier series of .

**Note: Recall Integration by parts rule.**

**EXISTANCE OF FOURIER SERIES:**

Functions that have Fourier series representation are those periodic functions which satisfy three Dirichlet conditions.

1. is absolutely integrable over one period. ()
2. has an infinite number of maxima and minima over one period. ()
3. had an infinite number of finite discontinuities over one period.

()

The periodic function equals to their Fourier series representation, except at some values of “x” where has finite discontinuity. At these values of “x”, the Fourier series converges to the average value of the function values on either side of the discontinuity.